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FIZEAU PLASMA INTERFEROMETER

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ABSTRACT

Plasma interferometers typically use Mach-Zender or Twyman-Green designs such that the fringe intensity $I(\delta) \propto \cos^2 \frac{\delta}{2}$ where δ is the phase shift. In situations where phase shifts are small, $\delta < \pi$, the interferometers are used in the region of maximum slope ($\delta = \pi/2$) where $\frac{dI}{d\delta} \propto I_o$. The Fizeau interferometer uses multiple reflections giving an intensity function where

$$I \propto \frac{f\sin^2 \frac{\delta}{2}}{1+f\sin^2 \frac{\delta}{2}},$$

thus the maximum $\frac{dI}{d\delta} \propto I_o\sqrt{f}$. Where f is a function of the reflectivity of the cavity ($f=4R/(1-R)^2$) and can reasonably have values in excess of 1000. Therefore with current detection techniques whereby $\Delta I/I$ can be measured to 10^{-2} or 10^{-3}, fringe shifts of 10^{-4} -10^{-5} can be measured. A fringe shift of 10^{-4} corresponds to a plasma line density of 2×10^{12} cm^{-2} for $\lambda = 10.6$ μm and 3.5×10^{13} for $\lambda = 638$ nm.

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This paper describes a technique by which the sensitivity of plasma interferometers can be increased. Stabilization and fractional fringe measurement techniques [1 2 3] have improved to the point where additional optical sensitivity could be useful.

The index of refraction for electromagnetic radiation passing through an ionized plasma is a function of the plasma density. Thus a measure of the index can be used to determine the plasma density.[4]

The index of refraction, n, in an ionized plasma for an electromagnetic wave of frequency ω is given by:[5]

$$n = \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \qquad (1)$$

where ω_p is the plasma frequency given by

$$\omega_p^2 = \frac{4\pi N e^2}{m} \qquad (2)$$

where N = electron density
m = electron mass
e = electron charge

Expression (1) can also be written,

$$n = \left(1 - \frac{N}{N_c}\right)^{1/2} \qquad (3)$$

where N_c is the critical density such that $\omega = \omega_p$ and the plasma becomes opaque.

The index of refraction can be determined by measuring the phase shift of a laser beam passing through the plasma with an interferometer. The phase shift, δ, is given by,[6]

$$\delta = \frac{2\pi}{\lambda} \int (n_0 - n)\, d\ell \qquad (4)$$

where λ is the vacuum wave length of the laser, and n_0 is the index of the region around the plasma. The integral is evaluated over the laser path. In the Mach-Zender interferometer (Fig. 1), where the plasma is in a vacuum, the phase shift becomes,

$$\delta = \frac{2\pi}{\lambda}(1-n)\ell$$

where ℓ is the diameter of the plasma. The phase shift of a Michelson interferometer (Fig. 2) is twice that for the Mach-Zender because the light passes twice through the plasma.

The intensity in the output plane of a two beam interferometer is,

$$I = 2I_0 \cos^2\frac{\delta}{2}$$

where I_0 is the input laser intensity split equally into the two beams. The sensitivity is dependent on the slope of the intensity, or

$$\frac{dI}{d\delta} = I_0 \cos\frac{\delta}{2} \sin\frac{\delta}{2}$$

when evaluated at $\delta = \pi/2$ maximum sensitivity is obtained:

$$\left.\frac{\Delta I}{I_o}\right|_{max} = \frac{1}{2}\Delta\delta$$

The Fizeau and Fabry-Perot interferometers are based on multiple reflections between two parallel partly reflecting plates (Fig. 3). For each member of either the reflected or transmitted set of waves the phase of the wave function differs from that of the preceding wave by twice the transit between the plates. Thus the plasma dependent phase shift is just that of the Michelson or,

$$\delta = \frac{4\pi}{\lambda}(1-n)\ell$$

The intensity of the light reflected(I_r)or transmitted(I_t)is found by summing the contributions of each pass through the interferometer. Requiring the reflectivity (R) of both plates to be equal and neglecting absorption, the intensities become,

$$I_r = \frac{I_o\, f \sin^2 \frac{\delta}{2}}{1 + f \sin^2 \frac{\delta}{2}} \qquad \text{(Fizeau Interferometer)}$$

$$I_t = \frac{I_o}{1 + f \sin^2 \frac{\delta}{2}} \qquad \text{(Fabry-Perot Interferometer)}$$

where $f = \frac{4R}{\left(1 - R\right)^2}$

The two functions are complementary with the Fizeau characterized by a bright field with sharp black fringes and the Fabry-Perot a dark field with sharp bright fringes.

The maximum sensitivity for either interferometer with phase shifts small compared with $\pi/2$, is, to the first order,

$$\left|\frac{\Delta I}{I_o}\right|_{max} = \frac{\sqrt{f}}{2}\left|\Delta\, \delta\right|$$

The sharpness of the fringes is usually measured by the ratio of adjacent fringes to the half width (FWHM). This ratio called the finesse (F) is related to F by,

$$F = \frac{\pi\sqrt{f}}{2}$$

Thus the maximum sensitivity becomes,

$$\left|\frac{\Delta I}{I_o}\right|_{max} = \frac{F}{\pi}\ \Delta\delta,$$

that is the intensity through the interferometer will change by a factor of 2 for a fringe shift of 1/2F.

One fringe corresponds to a plasma line density 3.5 x $10^{17}/cm^2$ for a HeNe laser at λ = 632.8 nm and 2.1 x $10^{16}/cm^2$ for a CO_2 laser at λ = 10.6 μm. With high reflectivity coatings, ultra flat and smooth surfaces, using a small diameter laser beam, a finesse in excess of 100 is achievable.

Thus, the half intensity shift corresponds to 1.8 x $10^{16}/cm^2$ for HeNe and $10^{14}/cm^2$ for CO_2. With techniques for measuring $\Delta I/I$ as small as 10^{-2} or 10^{-3} plasma density measurements are possible in the 10^{13} - $10^{14}/cm^2$ range for HeNe and $10^{12}/cm^2$ for CO_2.

There are several limiting design factors for multipass interferometers. The cavity fill time is a basic limit on the temporal resolution. Laser coherence length, mechanical and electronic stability and quality of optical components impose real, but surmountable, limits.

Figure 1 MACH ZENDER INTERFEROMETER

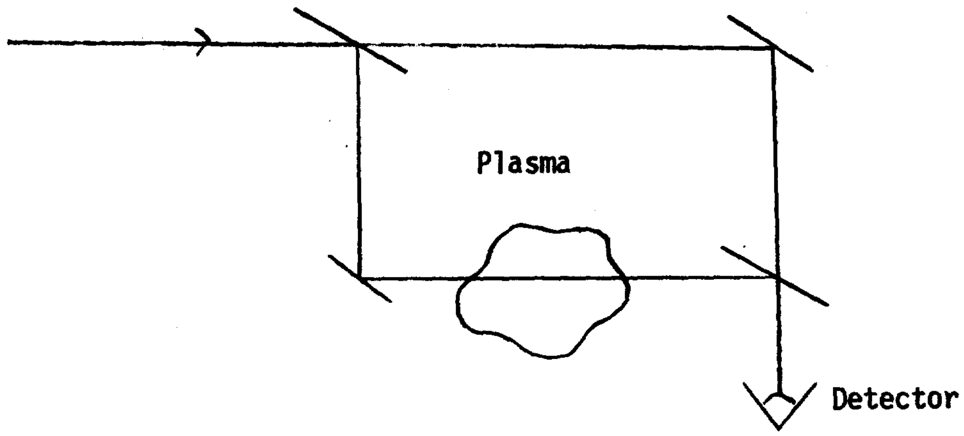


Figure 2 MICHELSON INTERFEROMETER

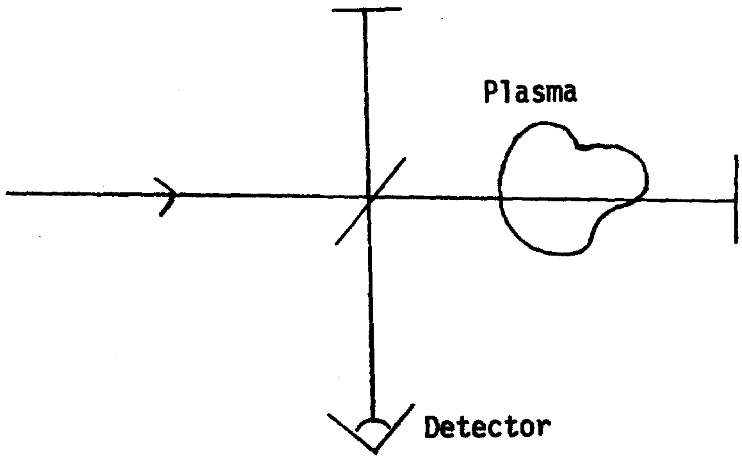
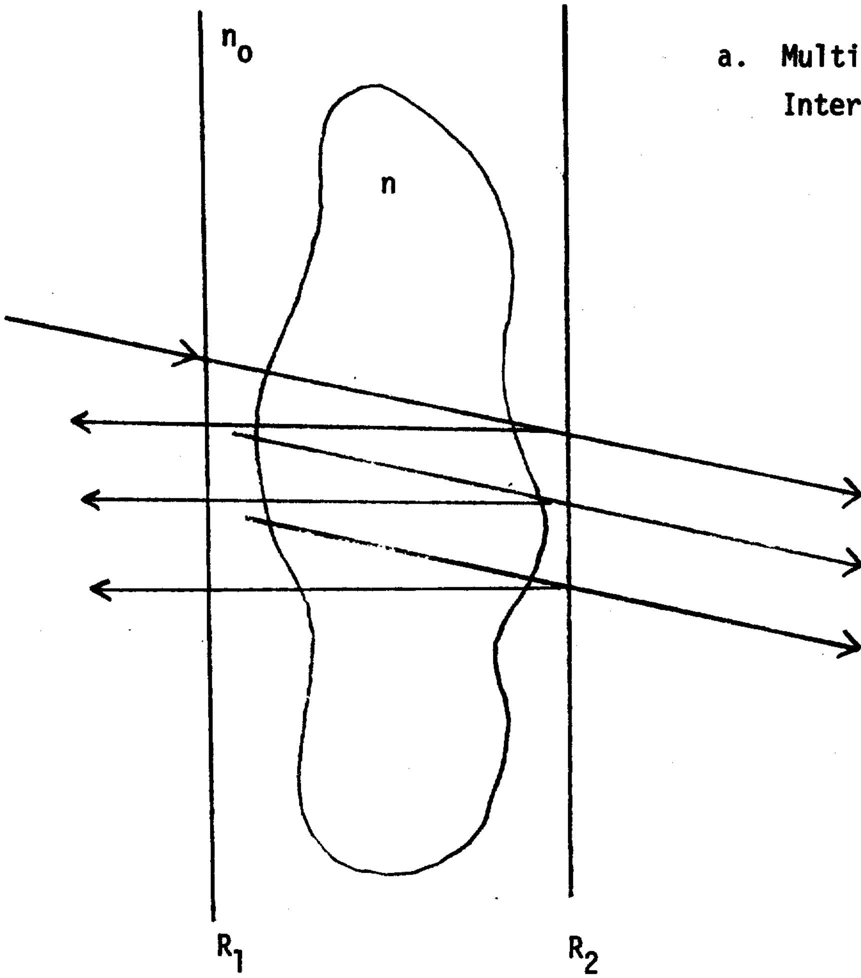
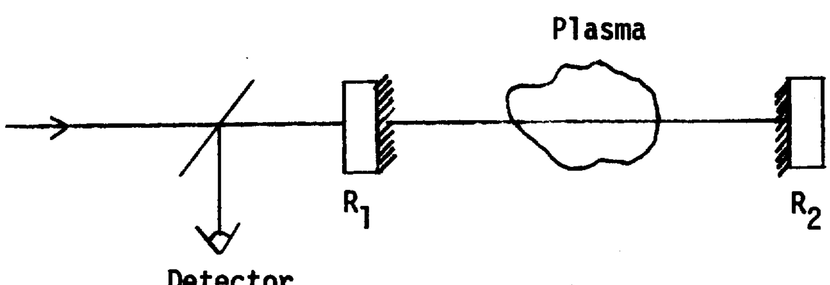


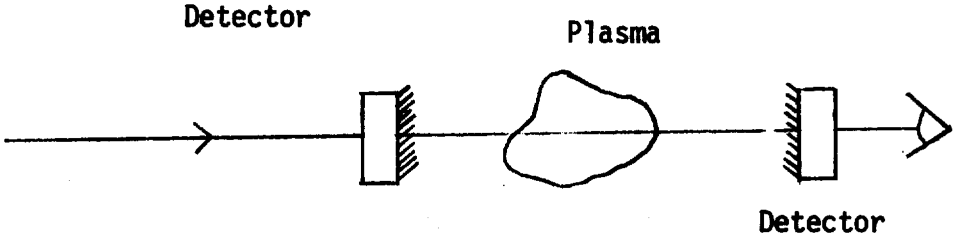
Figure 3



a. Multiple Reflection Interferometer



b. Fizeau Interferometer



c. Fabry-Perot Interferometer

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